About the Q^2 dependence of asymmetry A_1

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Abstract

We analyse the proton and deutron data on spin dependent asymmetry $A_1(x, Q^2)$ supposing the DIS structure functions $g_1(x, Q^2)$ and $F_3(x, Q^2)$ have the similar Q^2-dependence. As a result, we have obtained $\Gamma_1^p - \Gamma_1^n = 0.192$ at $Q^2 = 10$ GeV^2 and $\Gamma_1^p - \Gamma_1^n = 0.165$ at $Q^2 = 3$ GeV^2, in the best agreement with the Bjorken sum rule predictions.

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An experimental study of the nucleon spin structure is realized by measuring of the asymmetry $A_1(x,Q^2) = g_1(x,Q^2)/F_1(x,Q^2)$. The most known theoretical predictions on spin dependent structure function $g_1(x,Q^2)$ of the nucleon were done by Bjorken [1] and Ellis and Jaffe [2] for the so called *first moment value* $\Gamma_1 = \int_0^1 g_1(x)dx$.
The calculation of the Γ_1 value requires the knowledge of structure function g_1 at the same Q^2 in the hole x range. Experimentally asymmetry A_1 is measuring at different values of Q^2 for different x bins. An accuracy of the past and modern experiments [3] - [9] allows to analyze data in the assumption [10]-[11] that asymmetry $A_1(x,Q^2)$ is Q^2 independent (structure functions g_1 and F_1 have the same Q^2 dependence). But the tune checking of the Bjorken and Ellis - Jaffe sum rules requires considering the Q^2 dependence of A_1 or g_1 (for recent studies of the Q^2 dependence of A_1 see [10]-[15])

This article is based on our observation[3] that the Q^2 dependence of g_1 and the spin average structure function F_3 is the same in a wide x range: $10^{-2} < x < 1$. At small x it seems that may be not true (see [16], [17]-[20]).

To demonstrate the validity of the observation, lets consider the nonsinglet (NS) Q^2 evolution of structure functions F_1, g_1 and F_3. The DGLAP equation for the NS part of these functions can be presented as[4] :

$$\begin{aligned}
\frac{dg_1^{NS}(x,Q^2)}{dlnQ^2} &= -\frac{1}{2}\gamma^-_{NS}(x,\alpha) \times g_1^{NS}(x,Q^2), \\
\frac{dF_1^{NS}(x,Q^2)}{dlnQ^2} &= -\frac{1}{2}\gamma^+_{NS}(x,\alpha) \times F_1^{NS}(x,Q^2), \qquad (1)\\
\frac{dF_3^{NS}(x,Q^2)}{dlnQ^2} &= -\frac{1}{2}\gamma^-_{NS}(x,\alpha) \times F_3^{NS}(x,Q^2),
\end{aligned}$$

where symbol $\times$ means the Mellin convolution. Functions $\gamma^\pm_{NS}$ are the reverse Mellin transforms of the anomalous dimensions $\gamma^\pm_{NS}(n,\alpha) = \alpha\gamma^{(0)}(n)_{NS} + \alpha^2\gamma^{\pm(1)}_{NS}(n) + O(\alpha^3)$ and the Wilson coefficients[5] $\alpha b^\pm(n) + O(\alpha^2)$:

$$\gamma^\pm_{NS}(x,\alpha) \;=\; \alpha\gamma^{(0)}_{NS}(x) + \alpha^2\Big(\gamma^{\pm(1)}_{NS}(x) + 2\beta_0 b^\pm(x)\Big) + O(\alpha^3), \qquad (2)$$

where $\beta(\alpha) = -\alpha^2\beta_0 - \alpha^3\beta_1 + O(\alpha^4)$ is QCD β-function.
The above mentioned Mellin transforms mean that

$$f(n,Q^2) \;=\; \int_0^1 dx x^{n-1} f(x,Q^2), \qquad (3)$$

where $f = \{\gamma^{(0)}_{NS}, \gamma^{\pm(1)}_{NS}, b^\pm_{NS}, \gamma^{(k)}_{ij}, \gamma^{*(k)}_{ij}, b_i \text{ and } b^*_i\}$ with $k = 1,2$ and $\{i,j\} = \{S,G\}$.

[3]The conclusion connects with our previous analysis [14].

[4]We use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

[5]Because we consider here the structure functions themselves but not the quark distributions. Note that more standard definition of $b^+_{NS}(n)$ and $b^-_{NS}(n)$ are $b_{1,NS}(n) = b_{2,NS}(n) - b_{L,NS}(n)$ and $b_{3,NS}(n)$.

Eqs. (1) show the Q^2 dependence of NS parts of g_1 and F_3 is the same (at least in first two orders of the perturbative QCD [21]) and differs from F_1 already in the first subleading order ($\gamma_{NS}^{+(1)} \neq \gamma_{NS}^{-(1)}$ [22] and $b_{NS}^{+} - b_{NS}^{-} = (8/3)x(1-x)$). For the singlet parts of g_1 and F_1 evolution equations are :

$$\begin{aligned} \frac{dg_1^S(x,Q^2)}{dlnQ^2} &= -\tfrac{1}{2}\Big[\gamma_{SS}^*(x,\alpha) \times g_1^S(x,Q^2) + \gamma_{SG}^*(x,\alpha) \times \Delta G(x,Q^2)\Big], \\ \frac{dF_1^S(x,Q^2)}{dlnQ^2} &= -\tfrac{1}{2}\Big[\gamma_{SS}(x,\alpha) \times F_1^S(x,Q^2) + \gamma_{SG}(x,\alpha) \times G(x,Q^2)\Big], \end{aligned} \tag{4}$$

where

$$\begin{aligned} \gamma_{SS}(x,\alpha) &= \alpha\gamma_{SS}^{(0)}(x) + \alpha^2\Big(\gamma_{SS}^{(1)}(x) + b_G(x) \times \gamma_{GS}^{(0)}(x) + 2\beta_0 b_S(x)\Big) \\ &\qquad + O(\alpha^3), \\ \gamma_{SG}(x,\alpha) &= \frac{e}{f}\Big[\alpha\gamma_{SG}^{(0)}(x) + \alpha^2\Big(\gamma_{SG}^{(1)}(x) + b_G(x) \times (\gamma_{GG}^{(0)}(x) - \gamma_{SS}^{(0)}(x)) + 2\beta_0 b_G(x) \\ &\qquad + b_S(x) \times \gamma_{SG}^{(0)}(x)\Big)\Big] + O(\alpha^3) \end{aligned}$$

where $e = \sum_i^f e_i^2$ is the sum of charge squares of f active quarks. The equations for polarized anomalous dimensions $\gamma_{SS}^*(x,\alpha)$ and $\gamma_{SG}^*(x,\alpha)$ are similar. They ban be obtained by replacing $\gamma_{Si}^{(1)}(x) \rightarrow \gamma_{Si}^{*(1)}(x)$ and $b_i(x) \rightarrow b_i^*(x)$ $(i = \{S,G\})$.

Note here the gluon term is not negligible for F_1 at $x < 0.1$ but for g_1 we can neglect the gluons for $x > 0.03$ [13]-[17]. The value $b_s^*(x)$ ($b_s(x)$) coincides with $b^-(x)$ ($b^+(x)$). The difference between $\gamma_{NS}^{-(1)}$ and $\gamma_{SS}^{*(1)} + b_G^*(x) \times \gamma_{GS}^{(0)}(x)$ is negligible due to its difference having no a power singularity at $x \rightarrow 0$ (i.e. no a singularity for them momentum transforms at $n \rightarrow 1$ in momentum space) and decreases as $O(1-x)$ at $x \rightarrow 1$ [26] (see also [27]). Contrary to this, the difference between $\gamma_{SS}^{(1)} + b_G(x) \times \gamma_{GS}^{(0)}(x)$ and $\gamma_{SS}^{*(1)} + b_G^*(x) \times \gamma_{GS}^{(0)}(x)$ contains the power singularity at $x \rightarrow 0$ (see [28, 21]).
This observation allows us to conclude the function :

$$A_1^*(x) = \frac{g_1(x,Q^2)}{F_3(x,Q^2)} \tag{5}$$

should be practically Q^2 independent at $x > 0.01$. Because the r.h.s. of Eqs.(1) and (4) contain integrals of structure functions, the approximate validity of (5) is supported also by the same x-dependence of $g_1(x,Q^2)$ and $F_3(x,Q^2)$ at fixed Q^2. The asymmetry A_1 at $Q^2 =< Q^2 >$ can be defined than as :

$$A_1(x_i, <Q^2>) = \frac{F_3(x_i, <Q^2>)}{F_3(x_i, Q_i^2)} \cdot \frac{F_1(x_i, Q_i^2)}{F_1(x_i, <Q^2>)} \cdot A_1(x_i, Q_i^2), \tag{6}$$

where x_i (Q_i^2) means an experimentally measured value of x (Q^2).

We use SMC and E143 proton and deuteron data for asymmetry $A_1(x,Q^2)$ [6] - [9]. To get $F_1(x,Q^2)$ we take NMC parametrization for $F_2(x,Q^2)$ [23] and SLAC parametrization for $R(x,Q^2)$ [24] ($F_1 \equiv F_2/2x[1+R]$). To get the values of $F_3(x,Q^2)$ we parametrize the CCFR data [25] as a function of x and Q^2 (see Fig.1).
First, using eq.5, we recalculate the SMC [6, 7] and E143 [8, 9] measured asymmetry of the proton and deuteron at $Q^2 = 10$ GeV^2 and 3 GeV^2, which are average Q^2 of these experiments respectively (results are shown in Fig.2, 3) and get the value of $\int g_1(x)dx$ through the measured x ranges (see Table 1).
To obtain the first moment values $\Gamma_1^{p(d)}$ we have used an original estimations of SMC and E143 for unmeasured regions [6] - [9]. Results on the Γ_1 values are shown in the Table 1.

Table 1. The first moment value of g_1 of the proton and deuteron.

$x_{min} - -x_{max}$	$< Q^2 >$	target type	$\int_{x_{min}}^{x_{max}} g_1 dx$	Γ_1	experiment
$.003 - 0.7$	10 GeV^2	proton	0.130	0.135	SMC
$.003 - 0.7$	10 GeV^2	deuteron	0.038	0.0362	SMC
$.029 - 0.8$	3 GeV^2	proton	0.123	0.130	E143
$.029 - 0.8$	3 GeV^2	deuteron	0.043	0.044	E143

As the last step we calculate the difference $\Gamma_1^p - \Gamma_1^n = 2\Gamma_1^p - (\Gamma_1^p + \Gamma_1^n)$ where $\Gamma_1^p + \Gamma_1^n = 2\Gamma_1^d/(1 - 1.5 \cdot \omega_D)$ and $\omega_D = 0.05$ [7, 9] .
At $Q^2 = 10$ GeV^2 we get the following results :

$$\begin{aligned}
\Gamma_1^p - \Gamma_1^n &= 0.199 \pm 0.038 && \text{(SMC [7])} \\
\Gamma_1^p - \Gamma_1^n &= 0.192 && \text{(our result)} \\
\Gamma_1^p - \Gamma_1^n &= 0.187 \pm 0.003 && \text{(Theory)}
\end{aligned} \tag{7}$$

and at $Q^2 = 3$ GeV^2 :

$$\begin{aligned}
\Gamma_1^p - \Gamma_1^n &= 0.163 \pm 0.026 && \text{(E143 [9])} \\
\Gamma_1^p - \Gamma_1^n &= 0.165 && \text{(our result)} \\
\Gamma_1^p - \Gamma_1^n &= 0.171 \pm 0.008 && \text{(Theory)}
\end{aligned} \tag{8}$$

As a conclusion, we would like to note

- our observation that function $A_1^*(x)$ is Q^2 independent at large and intermediate x is supported by good agreement (see Fig. 2,3) of present analysis with other estimations [12]-[16] of the Q^2 dependence of the A_1;

- at small x structure functions $g_1(x,Q^2)$ and $F_3(x,Q^2)$ may have the same behaviour too (in traditional, Regge-motivated consideration $f \sim x^\delta$, where $\delta \geq 0$)) [30] [6].

[6] According to the recent analysis [17]-[20], however, the situation may be more complicated.

- The value of $\Gamma_1^p - \Gamma_1^n$ obtained in the supposion that g_1 and F_3 have the same Q^2 dependence improves the agreement with the Bjorken sum rule prediction.

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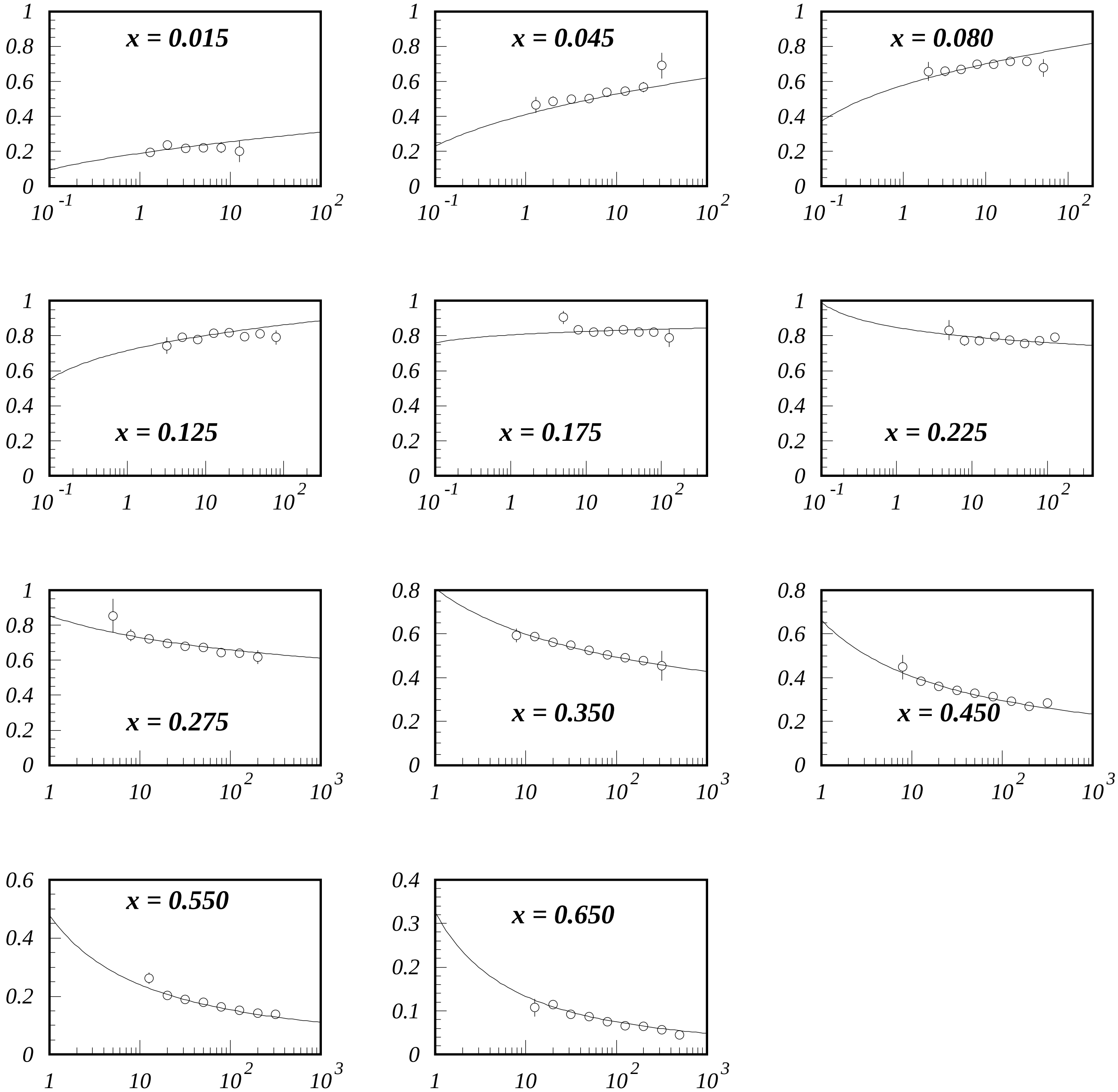
Figure Captions

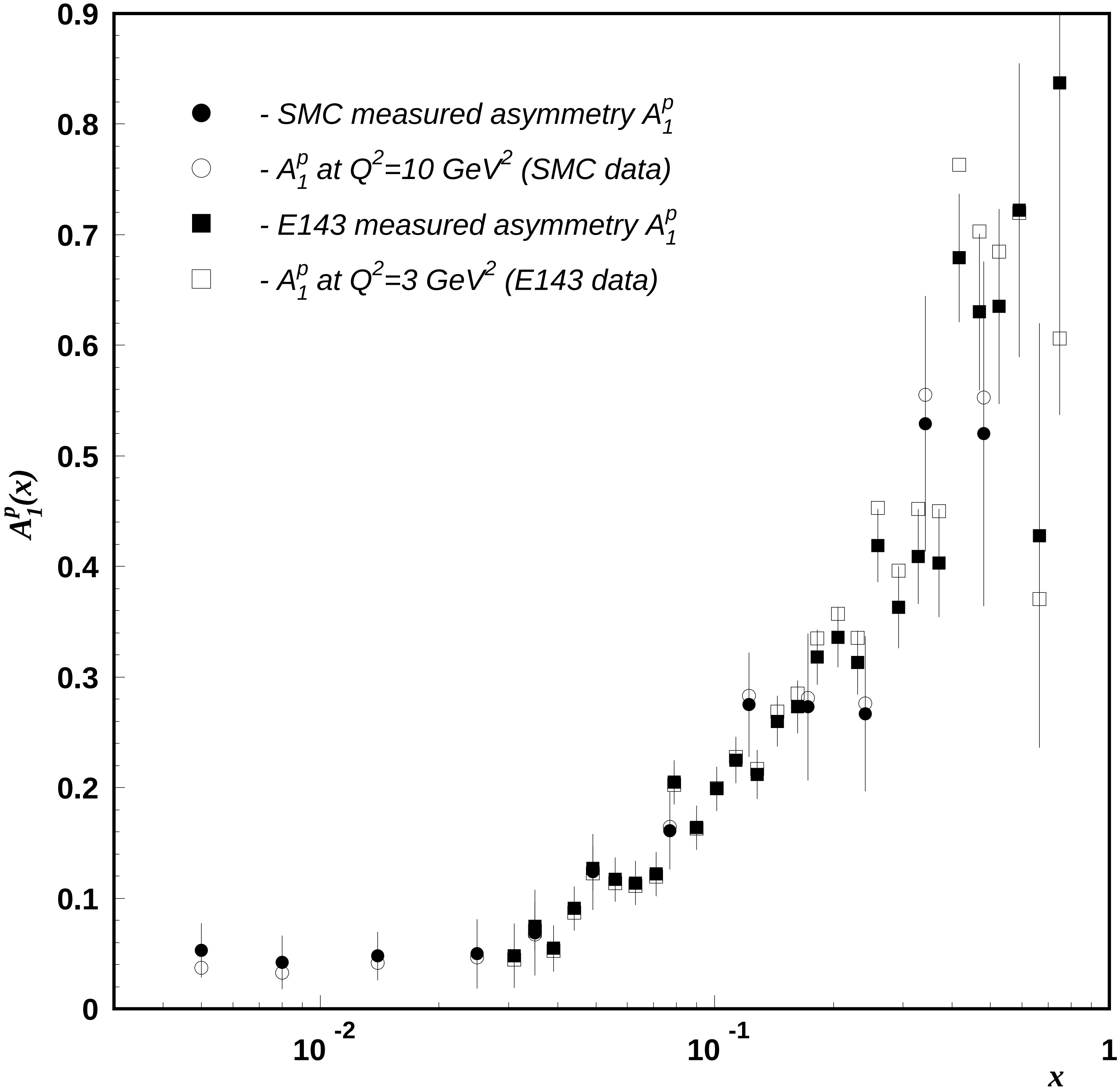
Figure 1. DIS structure function $F_3(x, Q^2)$. CCFR data [25] and the parametrization.

Figure 2. SMC [6] and E143 [8] measured virtual photon-nucleon asymmetry $A_1^p(x)$ as a function of x (shown as a close points) in comparison with evolved to $Q^2 = 10$ (3) GeV^2, respectively (open points).

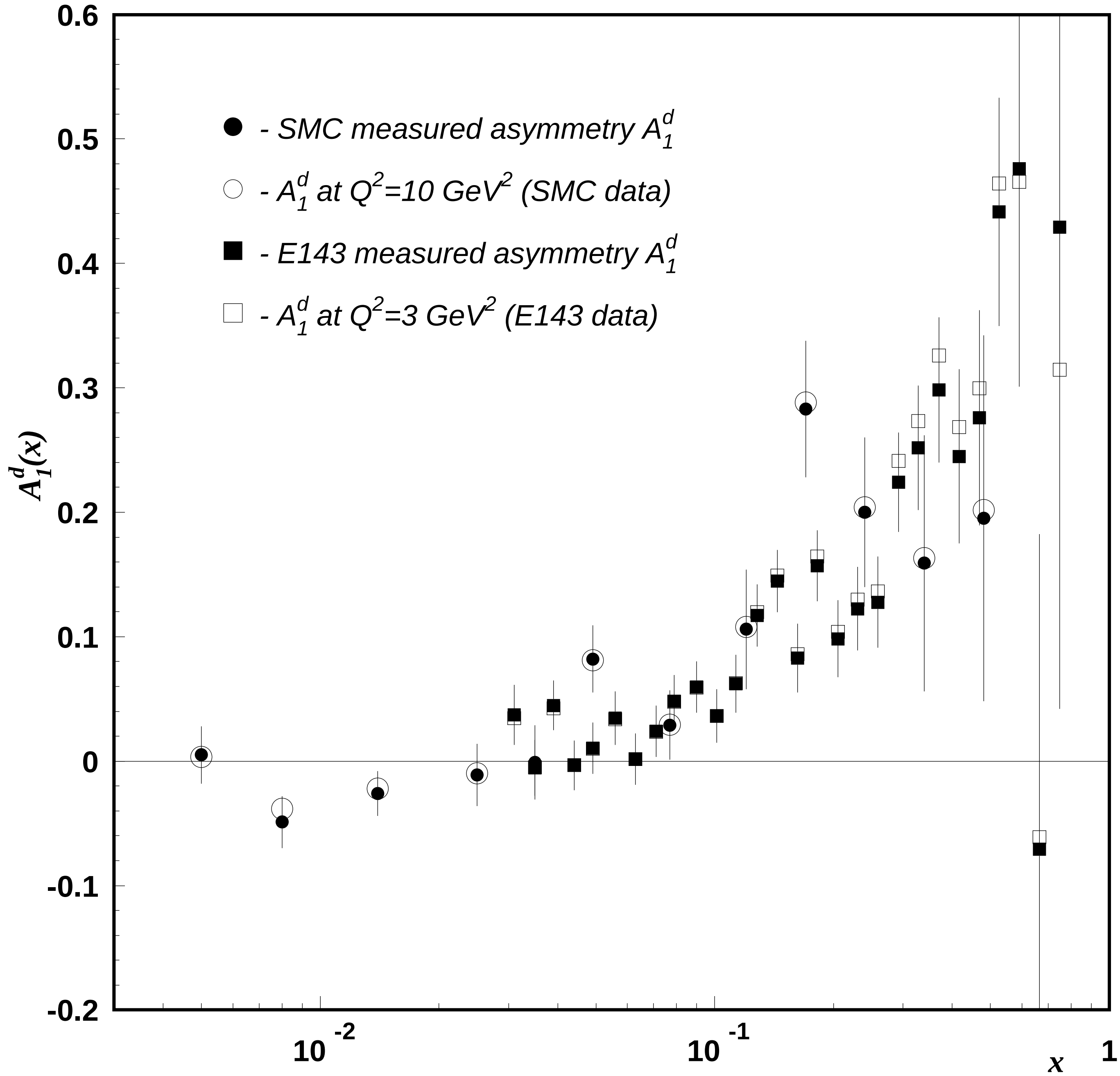
Figure 3. SMC [7] and E143 [9] measured virtual photon-nucleon asymmetry $A_1^d(x)$ as a function of x (shown as a close points) in comparison with evolved to $Q^2 = 10$ (3) GeV^2, respectively (open points).



x = 0.015
x = 0.045
x = 0.080
x = 0.125
x = 0.175
x = 0.225
x = 0.275
x = 0.350
x = 0.450
x = 0.550
x = 0.650



- SMC measured asymmetry A_1^p
- A_1^p at Q^2=10 GeV^2 (SMC data)
- E143 measured asymmetry A_1^p
- A_1^p at Q^2=3 GeV^2 (E143 data)
$A_1^p(x)$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^{-2}
10^{-1}
1
x



- SMC measured asymmetry A_1^d
- A_1^d at Q^2=10 GeV^2 (SMC data)
- E143 measured asymmetry A_1^d
- A_1^d at Q^2=3 GeV^2 (E143 data)
$A_1^d(x)$
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
10^{-2}
10^{-1}
1
x